Motion

CK-12 Study Guides
MADE BY STUDENTS!

Physics

Big Picture

Everything is in motion. For example, we are all moving around the sun. This means that when we talk about motion, we must look at it relative to something else. The motion of objects through space is one of the first subjects of study for early physicists, but it took a very long time before motion was fully understood. To describe motion, we use rates such a velocity, speed, and acceleration. Throughout this study guide, assume negligible air resistance.

Key Terms

Displacement: The distance an object has moved from its starting position. SI units: m

Speed: The rate at which an object covers distance (distance/time). SI units: m/s

Velocity: Speed in a given direction. SI units: m/s

Acceleration: The rate at which an object's velocity is changing (velocity/time). Like velocity, acceleration is a vector. SI units: m/s^2

Projectile Motion: The non-linear path an object takes when moving under the constant acceleration of gravity.

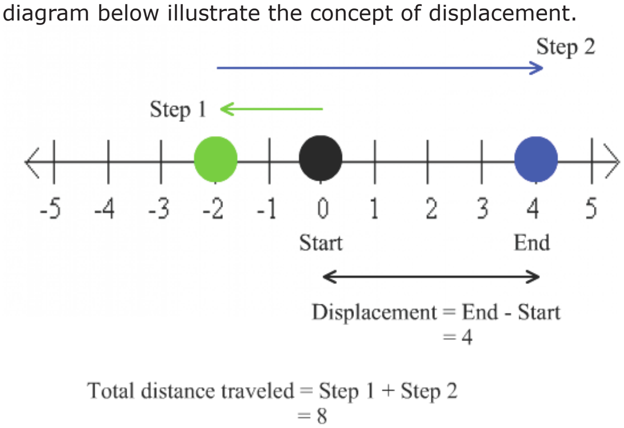
Free Fall: Objects falling due to gravity only. Example: an apple falling from a tree is in free fall.

Describing Motion

Displacement vs Distance

Displacement and distance both measure a change in position. Displacement is a vector quantity, while distance is a scalar.

If an object moves in one direction and then returns to its original position, the total distance will be a positive number and the total displacement will be 0. The diagram below illustrate the concept of displacement.



Speed vs Velocity

Speed is a scalar quantity that measures how fast an object is moving. In comparison, **velocity** is a vector quantity. Velocity is speed with a direction. Objects moving at the same speed can have different velocities if they are moving in different directions.

For example, we can say a car is moving at the speed of 25 mph, but if we say the car is moving at 25 mph to the north, then we are stating its velocity.

Acceleration

Usually we use "**acceleration**" to mean speeding up and "deceleration" to mean slowing down. In physics, acceleration is a vector quantity that could refer to speeding up, slowing down, or even changing direction.

Careful: positive acceleration does not always mean speeding up and negative acceleration does not always means slowing down.

Think of an acceleration as a push. We can define positive acceleration as a push to the right and negative acceleration as a push to the left.

If an object has a negative velocity, meaning that the object is moving towards the left, then a positive acceleration pushing the object to the right will cause the object to slow down.

- If the velocity and acceleration are in the same direction, the object speeds up. If not, the object slows down.

Instantaneous vs Average

There are two types of speed, velocity, and acceleration: instantaneous and average.

Instantaneous refers to a single moment in time, while average refers to over a period of time.

Notes

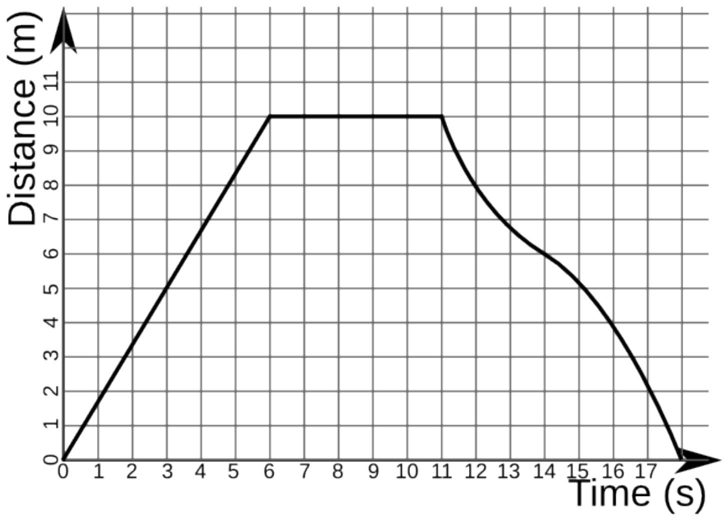
This guide was created by Christopher Addiego and Catherine Wu. To learn more about the student authors, http://www.ck12.org/about/ck-12-interns/.

Motion cont.

Graphs of Motion

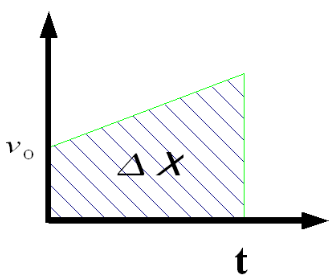
Distance vs. Time

Below is a graph of distance versus time. The slope $\left(\frac{\Delta y}{\Delta x}\right)$ of the curve represents the speed. As we can see, until t = 6 s, the object is moving at a constant speed of $\frac{5\text{ m}}{3\text{ s}}$. After t = 6 s, its speed is 0, meaning the object is at rest. Then, at 11 s, the object starts to move in the opposite direction. After 17 s, it has returned to its original position, resulting in a total displacement of 0.



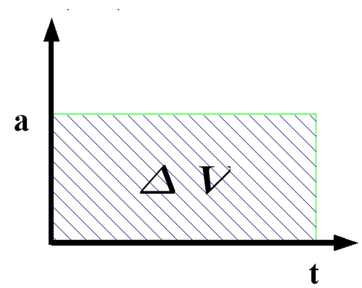
Velocity vs. Time

Below is a graph of velocity versus time. The slope represents acceleration. If the velocity curve is a line (has a constant slope), we know that there is a constant acceleration. The area under the velocity curve is equal to the change in displacement.



Acceleration vs. Time

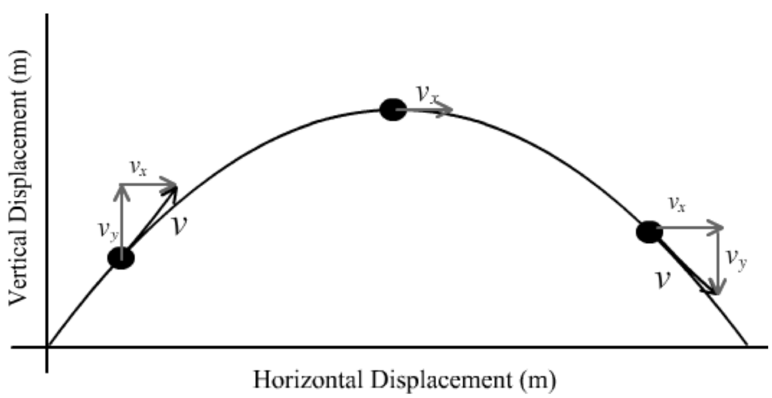
Similarly, for acceleration, the area under the acceleration curve is equal to the change in velocity. Below is a graph of acceleration versus time.



Projectile Motion

Projectile motion is a type of two-dimensional motion where an object moves horizontally (in the *x*-axis) and vertically (in the *y*-axis). The *x*- and *y*-components of any object's velocity vector are completely independent of each other. The diagram on the right shows the path and velocity vectors of an object in projectile motion.

- At the top of its flight, when the object stops rising and is about to fall, the vertical speed of an object is zero.
- Air resistance is ignored, so the horizontal component of velocity stays constant.
- The vertical component of velocity does not stay constant due to gravity - the object is in **free fall** along the vertical direction.



Motion Problem Guide

Physics

General Guidelines

- Always define a coordinate system for the problem! Define the positive and negative directions.
- Make sure your units make sense.

Important Equations

- All equations are only valid for constant acceleration
- In one dimension, vectors can only point in two directions, typically labeled + and -. The one dimensional vectors below can be treated like scalars with the signs indicating direction.

$v_{avg} = \frac{\Delta x}{\Delta t}$	x - displacement
$a_{avg} = \frac{\Delta v}{\Delta t}$	v - velociy
$v = v_o + at$	v_o - initial velocity
$\Delta x = v_o t + \frac{1}{2}at^2$	a - acceleration
$v^2 = v_o{}^2 + 2a\Delta x$	t - time

Δ - change in (final value - initial value)

Sample Problem

Two cities are 33 miles apart by freeway, where the speed limit is 65 mph. If it took you 45 minutes to get from one city to the other, what was your average speed?

Solution

Since we know the distance and the time, we can easily find the speed, which is just distance over time.

$$\text{speed} = \frac{\text{total distance}}{t}$$

$$\text{speed} = \frac{33 \text{ mi}}{.75 \text{ hr}}$$

$$\text{speed} = 44 \text{ mph}$$

Free Fall

An object thrown into the air is in free fall when it is moving up and falling down. At the maximum height, the object's velocity is 0 as it changes direction. If the object is at the same height before and after it is thrown, the time it takes to reach the maximum height is exactly half the total time the object spends in the air. The path up and the path down is identical but in opposite directions. Acceleration is always 9.8 m/s^2.

Often times, when we solve free fall problems, we look at only one direction (either the journey up or the journey down) because we know the velocity at the top is 0, meaning there is one less variable to deal with.

Equations to use: $v = v_o + at$, $v^2 = v_o^2 + 2a\Delta y$

Example

A tennis ball is hit up into the air and spends a total of 8 seconds in the air. How high does it reach before coming back down?

Solution

1. Determine what is given and what needs to be found.
2. Set up the equation $v = v_o + at$ and solve for v_o.
3. Set up the equation $v^2 = v_o^2 + 2a\Delta y$.
4. Plug in values and solve for y!

Given: t = 8 seconds
Find: y = ??

Let +y be upward and -y be the downward direction. In this example, we're going to use the ball on its way up, so t = 4 seconds, v = 0 m/s, and a = -9.8 m/s^2.

$v = v_o + at$	$v^2 = v_o^2 + 2ay$
$v_o = v - at$	$y = \frac{v^2 - v_o^2}{2a}$
$v_o = ??$	$y = ??$
$v_o = .8 \text{ m/s}^2 \cdot 4 \text{ s}$	$y = \frac{-(39.2 \text{ m/s})^2}{-2 \cdot 9.8 \text{ m/s}^2}$
$v_o = 39.2 \text{ m/s}$	$y = 78.4 \text{ m}$

So the ball reaches a height of 78 m.

Motion Problem Guide cont.

Projectile Motion

In projectile motion, the acceleration in the y-direction is always -9.8 m/s², while the acceleration in the x-direction is 0. If the object is thrown at an angle, the initial velocity must be broken up into x- and y-components ($v_x = v \cos\theta$, $v_y = v \sin\theta$). Like in freefall, the velocity in the y-direction at the top of the object's path is 0 m/s.

In some questions, the object is thrown at a specific angle "above the horizontal" or "below the horizontal." In the example and diagram below, you can see what "above the horizontal" means. "Below the horizontal" is similar, only v_y and acceleration are both in the same direction.

Equations to use: $v = v_o + at$

$$\Delta x = v_o t + \frac{1}{2}at^2$$

Example

A man standing on a cliff throws a rock at 25° above the horizontal with an initial velocity of 15 m/s. If the rock is in the air for 5 seconds, how tall is the cliff, and how far into the ocean does it travel?

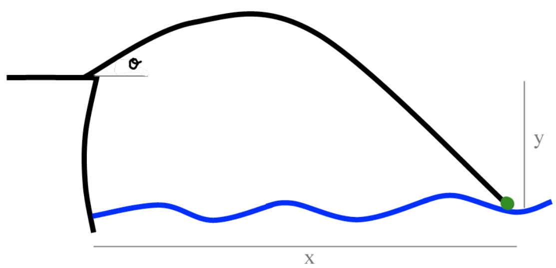
Solution

1. Determine what is given and what needs to be found.
2. Draw and label a diagram.
3. Set up the equation $\Delta x = v_o t + \frac{1}{2}at^2$ for the x-and y-directions.
4. Plug in values and solve!

Let the initial positions x_o and y_o be 0 so that $\Delta x = x - x_o = x$ and $\Delta y = y - y_o$.

Given: v_o = 15 m/s, t = 5 seconds, and θ = 15°

Find: x and y



x - direction

$$x = v_{o,x}t + \frac{1}{2}a_x t^2$$

$$v_{o,x} = v_o \cos\theta$$

$$a_x = 0 \text{ m/s}^2$$

$$x = 15 \text{ m/s} \cdot \cos(15^\circ)$$

$$x = 68.0 \text{ m}$$

y - direction

$$y = v_{o,y}t + \frac{1}{2}a_y t^2$$

$$v_{o,y} = v_o \sin\theta$$

$$a_y = -9.8 \text{ m/s}^2$$

$$y = 15 \text{ m/s} \cdot \sin(15^\circ) - \frac{1}{2} \cdot 9.8 \text{ m/s}^2 \cdot (5 \text{ s})^2$$

$$y = -90.8 \text{ m}$$

So the cliff is 91 m tall, and the rock travels 68 m.

In this problem, we end up with a negative y value that represents the height of the cliff because y = 0 is at the top of the cliff.

Example

If an object is launched into the air with speed at an angle on flat ground, how long will the object be in the air?

Solution

Since we know that the object's final vertical displacement is zero, we can think of its path as a parabola. We can find the roots using the displacement equation.

$\Delta y = v_o t + \frac{1}{2}at^2$	Start with the equation for vertical displacement.
$0 = v_y \sin\theta\, t - \frac{1}{2}gt^2$	Substitute in known values. The sign changed on the second term because we know g is negative.
$0 = t(v_y \sin\theta - \frac{1}{2}gt)$	Begin solving for t like we would for any quadratic.
$t = 0, \frac{2v_y \sin\theta}{g}$	We already know the displacement is 0 at time $t = 0$, so the second solution is our answer.